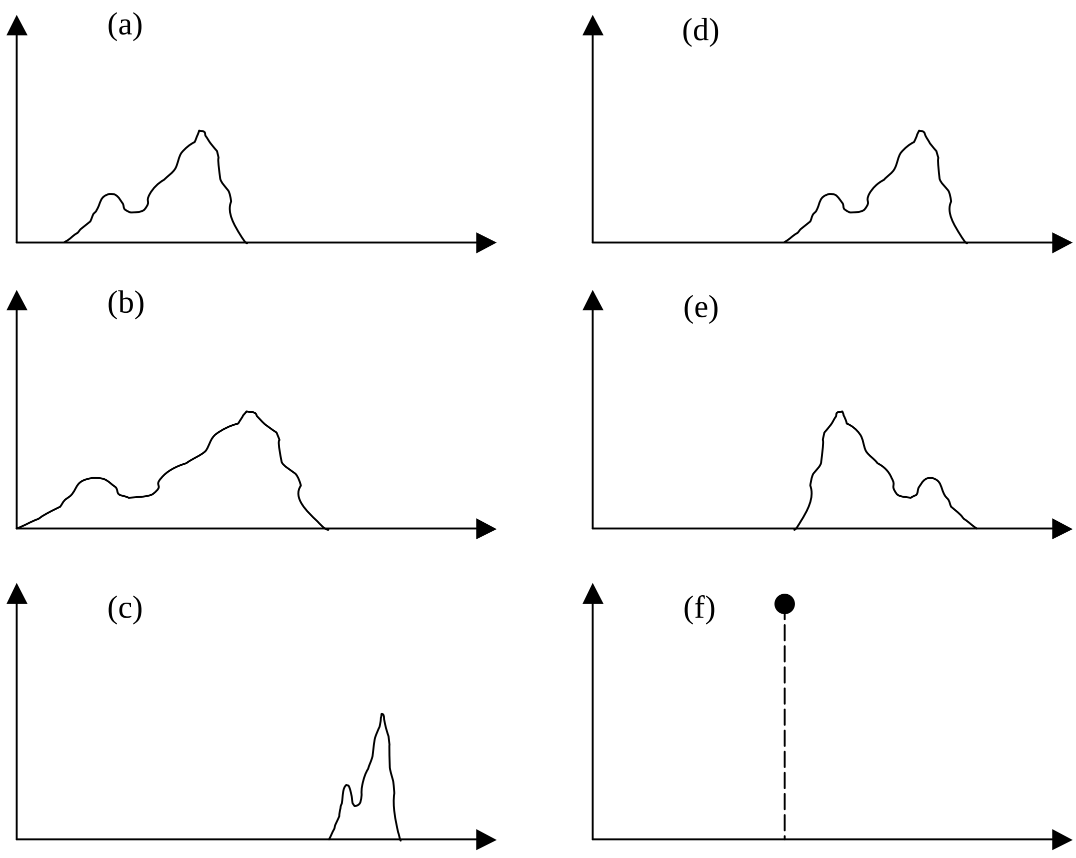
I

Figure (a) represents the gray level histogram of some image I and figures (b) to (f) represent the histograms of the images obtained after applying different gray level linear mappings to I. In each case, i.e., (b), (c), (d), (e) and (f): what can you say about the gain? what can you say about the bias? how is the brightness of the output image compared to the brightness of the input image I? how is the contrast of the output image compared to the contrast of I?



II

As mentioned in class, convolution is a bilinear operation. In particular,

$$h*(af_1+bf_2)=a(h*f_1)+b(h*f_2)$$

for any convolution kernel h, any two real numbers a and b and any two images f_1 and f_2.
Using the general formula of convolution, prove the equality above.

III

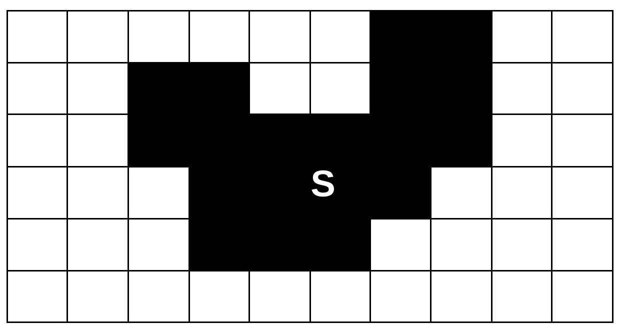
Give the definition of the histogram, the normalized histogram, the cumulative histogram and the normalized cumulative histogram of an image f. Describe in a sentence or two the principle of histogram equalization.

IV

In these images, the gray level 0 represents *false* and the gray level 255 represents *true*.
Draw up the truth table of the logic operator *OP* defined by: A *OP* B ≡ C.
Rewrite A *OP* B using only *NOT* and *OR*. Rewrite A *OP* B using only *NOT* and *AND*.
Many continuous functions from $\mathbb{R}^2$ to $\mathbb{R}$ can be used to implement *OP*. Give two examples.

V

Consider the chessboard distance *d* and the following image region *S*:



For any pixel *p*, the distance from *p* to *S* is $d_S(p) = min_{q \in S}\, d(p,q)$.
Compute $d_S(p)$ for any pixel *p* that does not belong to *S*.
Please use the grid above (i.e., fill in the white squares).

VI

Consider the image below. As an example, (e3,e4,e5) is a path from pixel e3 to pixel e5.

a1	a2	a3	a4	a5	a6	a7
b1	b2	b3	b4	b5	b6	b7
c1	c2	c3	c4	c5	c6	c7
d1	d2	d3	d4	d5	d6	d7
e1	e2	e3	e4	e5	e6	e7
f1	f2	f3	f4	f5	f6	f7
g1	g2	g3	g4	g5	g6	g7
h1	h2	h3	h4	h5	h6	h7
i1	i2	i3	i4	i5	i6	i7

1/ Is it possible to find a 4-path from e3 to e5, of length 10, and that goes through exactly 8 different pixels (including e3 and e5)?
2/ Is it possible to find a 4-path from e3 to e5, of length 2001?
3/ Is it possible to find an 8-path from e3 to e5, of length 10, that goes through exactly 8 different pixels and is not a 4-path?
4/ Is it possible to find an 8-path from e3 to e5, of length 2001, and that is not a 4-path?

If the answer to a question is yes, give an example of such a path. If the answer is no, explain.