I

The gray level mapping is a function m from $\Re$ into $\Re$ such that: $\forall u \in \Re$, $m(u)=au+b$.
a is the gain and b is the bias.

(b) $a > 1$ and $b < 0$. Better contrast. A bit brighter.
(c) $0 < a < 1$ and $b > 0$. Less contrast. Brighter.
(d) $a \approx 1$ and $b > 0$. Same contrast. Brighter.
(e) $a \approx -1$ and $b > 0$. Same contrast. Brighter.
This seems to be the histogram of the negative image.
(f) $a = 0$ and $b > 0$. No contrast. All pixels have the same gray level.

II

$$
\begin{aligned}
\forall (x,y),\quad [h*(af_1+bf_2)](x,y) &= \Sigma_i \Sigma_j h(i,j)\,(af_1+bf_2)(x-i,y-j) \\
&= \Sigma_i \Sigma_j h(i,j)\,[af_1(x-i,y-j)+bf_2(x-i,y-j)] \\
&= \Sigma_i \Sigma_j [ah(i,j)f_1(x-i,y-j)+bh(i,j)f_2(x-i,y-j)] \\
&= a\,\Sigma_i \Sigma_j h(i,j)f_1(x-i,y-j) + b\,\Sigma_i \Sigma_j h(i,j)f_2(x-i,y-j) \\
&= a(h*f)(x,y) + b(h*f)(x,y)
\end{aligned}
$$

Therefore: $h*(af_1+bf_2) = a(h*f_1) + b(h*f_2)$

III

Assume M×N is the size of f and L is the number of possible gray level values. The histogram of f is the function H_f from 0..L−1 into 0..M×N such that for any u in 0..L−1, the value $H_f(u)$ is the cardinal of the set $\{(x,y)\in(0..M-1)\times(0..N-1) \mid f(x,y)=u\}$. In other words, $H_f(u)$ is the number of pixels whose gray level is u. See your class notes for the definition of the other histograms. The principle of histogram equalization is to find a gray level mapping that yields an “optimal” improvement in contrast. Ideally, the histogram of the image after histogram equalization is flat. Note that the improvement is optimal *statistically*, not necessarily perceptually.

IV

A	B	A *OP* B
255 (*true*)	255 (*true*)	255 (*true*)
255 (*true*)	0 (*false*)	255 (*true*)
0 (*false*)	255 (*true*)	0 (*false*)
0 (*false*)	0 (*false*)	255 (*true*)

$$A \; OP \; B \equiv A \Leftarrow B$$
$$A \; OP \; B \equiv A \vee \neg B$$
$$A \; OP \; B \equiv \neg(\neg A \wedge B)$$

Let a be the logical value of proposition A and b the logical value of proposition B. We saw in class that we can assign to $\neg$A the value 255–a and we can assign to A$\vee$B the value *max*(a,b), or *min*(a+b,255), or a+b–ab/255, etc. Therefore, we can assign to A$\vee\neg$B, i.e., A *OP* B, the value *max*(a,255–b), or *min*(a+(255–b),255), or a+(255–b)–a(255–b)/255, etc. Two examples of functions that can be used to implement *OP* are:

$$f \mid (a,b) \mapsto max(a,255-b)$$
$$g \mid (a,b) \mapsto min(a+(255-b),255)$$

Note that f≠g. For example, f(100,100)=155 and f(100,100)=255.

V

2	1	1	1	1	1			1	2
2	1			1	1			1	2
2	1							1	2
2	1	1			S		1	1	2
2	2	1				1	1	2	2
3	2	1	1	1	1	1	2	2	3

VI

1/ Yes. Example: (e3,d3,c3,c4,c5,d5,d4,c4,c5,d5,e5).
2/ No. The length of a 4-path from e3 to e5 is necessarily even.
3/ Yes. Example: (e3,d3,c3,c4,c5,d5,c4,d4,c5,d5,e5).
4/ Yes. Example: (e3,d3,d4,e5,d4,e5, …, d4,e5), where "d4,e5," is repeated 1000 times.